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# DEVIATION OF THERMAL ANEMOMETER SENSORS WITH SAGGING WIRES FROM THE COSINE LAW

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The authors describe a calculation method and determine numerical values for the influence of the sag of the measuring wire of a thermal anemometer sensor on the deviation from a cosine law.

In determining the absolute magnitude of the velocity vector in three-dimensional flows using thermal anemometer sensor wires one measures the magntiude of the effective component of the flow velocity, which influences the heat tranfer between the wire and the flow. These quantities are related by the cosine law [1]:

$$V_\delta = V\cos\delta. \tag{1}$$

When one allows for the influence of the longitudinal velocity component on the heat transfer the cosine law takes the form [1]

$$V_\delta = V(\cos^2\delta + k^2\sin^2\delta)^{1/2}. \tag{2}$$

These relations are derived on the assumption that the measuring wire of the thermal anemometer sensor is straight. However, this condition does not hold in actual sensors. The deviation of the measuring wire from the straight condition stems from technical causes in the sensor manufacture, and also from the linear thermal expansion of the wire.

We now derive the relation between the magnitudes of the effective component and the flow velocity vector for the case of a sagging wire, when the wire forms the arc of a circle. The effective component of the velocity vector in the segment of arc QP of the measuring wire DQA (Fig. 1) varies from $V_{\delta_1}$ to $V_{\delta_2}$. The area of the figure FTQP is

$$S_{FTQP} = S_{OFT} - S_{OPQ}. \tag{3}$$

Using the notation

$$OP = OQ = r, \tag{4}$$

we obtain

$$PF = V_\delta t, \tag{5}$$

$$S_{FTQP} = \frac{1}{2}\int_{\delta_1}^{\delta_2}(r + V_\delta t)^2\,d\delta - \frac{1}{2}\int_{\delta_1}^{\delta_2} r^2 d\delta = \int_{\delta_1}^{\delta_2}\left(rV_\delta t + \frac{1}{2}V_\delta^2 t^2\right)d\delta. \tag{6}$$

The average value of the integrand is determined by the relation [2]

$$\left(rV_\delta t + \frac{1}{2}V_\delta^2 t^2\right)_{\text{cp}} = \frac{180}{(\delta_2 - \delta_1)\pi}\int_{\delta_1}^{\delta_2}\left(rV_\delta t + \frac{1}{2}V_\delta^2 t^2\right)d\delta. \tag{7}$$

Denoting the right side of Eq. (7) by I, we finally obtain

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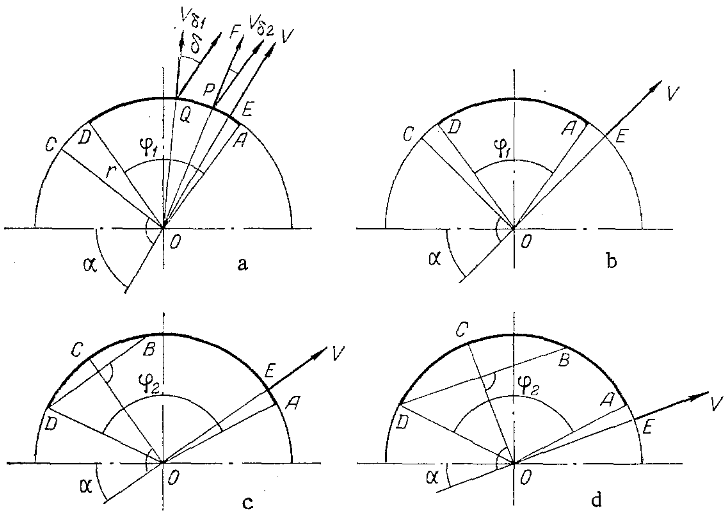


Fig. 1. Basic relative positions of the flow velocity vector and the measuring wire of the thermal anemometer for various angles $\alpha$ and $\varphi$, if: a) $\alpha \geqslant \varphi/2$, $\alpha \geqslant 90° - \varphi/2$ b) $\varphi/2 \leqslant \alpha \leqslant 90° - \varphi/2$; c) $90° - \varphi/2 \leqslant \alpha \leqslant \varphi/2$; d) $\alpha \leqslant 90° - \varphi/2$, $\alpha \leqslant \varphi/2$.

$$V_{\delta_{\text{av}}} = (-rt \pm \sqrt{r^2t^2 + 2It})\, t^{-2}. \tag{8}$$

We take the absolute value of the mean effective component of the velocity vector for a curved measuring wire to be the absolute magnitude of the effective component of the velocity vector for a straight measuring wire of the same length, if the heat transmitted by the two wires is the same. For the basic cases of relative position of the flow velocity vector and the measuring wire (Fig. 1), the quantity I is determined by the following expressions:

$$I_1 = \frac{180}{\varphi\pi} Vt \Big\{ r \Big( \int_0^{\delta_1} \sqrt{1-(1-k^2)\sin^2\delta}\, d\delta + \int_0^{\delta_2} \sqrt{1-(1-k^2)\sin^2\delta}\, d\delta \Big) + Vt \Big[ \frac{(\delta_1+\delta_2)\pi}{180}(k^2+1) - (\sin 2\delta_1 + \sin 2\delta_2)\frac{k^2-1}{2} \Big] \Big\}, \tag{9}$$

$$I_2 = \frac{180}{\varphi\pi} Vt \Big\{ r \int_{\delta_3}^{\delta_4} \sqrt{1-(1-k^2)\sin^2\delta}\, d\delta + Vt \Big[ \frac{(\delta_4-\delta_3)\pi}{180}(k^2+1) - (\sin 2\delta_4 - \sin 2\delta_3)\frac{k^2-1}{2} \Big] \Big\}, \tag{10}$$

$$I_3 = \frac{180}{\varphi\pi} Vt \Big\{ r \Big[ \int_0^{\delta_5} \sqrt{1-(1-k^2)\sin^2\delta}\, d\delta + \int_0^{\delta_6} \sqrt{1-(1-k^2)\sin^2\delta}\, d\delta + \int_{90}^{\delta_7} \sqrt{1-(1-k^2)\sin^2\delta}\, d\delta \Big] + Vt \Big[ \frac{(\delta_5+\delta_6+\delta_7-90)\pi}{180}(k^2+1) - (\sin 2\delta_5 + \sin 2\delta_6 + \sin 2\delta_7)\frac{k^2-1}{2} \Big] \Big\}, \tag{11}$$

$$I_4 = \frac{180}{\varphi\pi} Vt \Big\{ r \Big[ \int_{\delta_8}^{\delta_9} \sqrt{1-(1-k^2)\sin^2\delta}\, d\delta + \int_{90}^{\delta_{10}} \sqrt{1-(1-k^2)\sin^2\delta}\, d\delta \Big] + Vt \Big[ \frac{(\delta_8-\delta_9-\delta_{10}+90)\pi}{180}(k^2+1) - (\sin 2\delta_8 - \sin 2\delta_9 - \sin 2\delta_{10})\frac{k^2-1}{2} \Big] \Big\}, \tag{12}$$

where

$$\delta_1 = \alpha + \frac{\varphi_1}{2} - 90°, \tag{13}$$

$$\delta_2 = 90° - \alpha + \frac{\varphi_1}{2}, \tag{14}$$

$$\delta_3 = 90° - \alpha - \frac{\varphi_1}{2}, \tag{15}$$

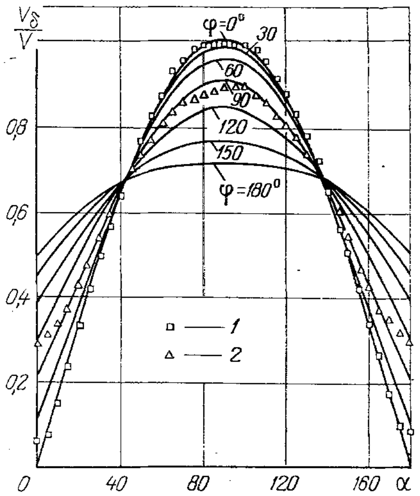


Fig. 2. Ratio of the effective component of the flow velocity vector to its absolute value $V_\delta/V$ as a function of the angle of attack $\alpha$ (deg) for circular arc-shaped thermal anemometer wires with different central angles $\varphi$, and experimental results: 1) for $\varphi = 0°$; 2) for $\varphi = 90°$.

$$\delta_4 = 90° - \alpha + \frac{\varphi_1}{2}, \tag{16}$$

$$\delta_5 = 90° + \alpha - \frac{\varphi_2}{2}, \tag{17}$$

$$\delta_6 = \alpha + \frac{\varphi_2}{2} - 90°, \tag{18}$$

$$\delta_7 = 90° - \alpha + \frac{\varphi_2}{2}, \tag{19}$$

$$\delta_8 = 90° - \alpha - \frac{\varphi_2}{2}, \tag{20}$$

$$\delta_9 = 90° + \alpha - \frac{\varphi_2}{2}, \tag{21}$$

$$\delta_{10} = 90° - \alpha + \frac{\varphi_2}{2}. \tag{22}$$

Since the sensitivity factor of the measuring wire has a value $0 \leqslant k \leqslant 0.3$, the integrals in Eqs. (9)-(12) are elliptic integrals of the second kind in Legendre form, for which numerical values are given in [3].

As can be seen from the relations (Fig. 2) calculated from Eqs. (8)-(22), a large sag in the measuring wire of the thermal anemometer sensor leads to a considerable deviation from the cosine law and to a reduction in sensitivity, which it is important to allow for in measuring the velocity vector of three-dimensional flows. A certain amount of discrepancy between the experimental results and the calculated curves (Fig. 2) can be explained by the deviation of the actual curved sensor wire from the arc of a circle.

## NOTATION

$V$, absolute magnitude of the flow velocity vector; $V_\delta$, absolute magnitude of the effective component of the flow velocity vector; $\alpha$, angle of attack; $\delta$, angle between the flow velocity vector and the normal to the wire; $\varphi$, $\varphi_1$, $\varphi_2$, central angle of an arc of a circle; $r$, radius of a circular arc; $k$, sensitivity factor of the sensor wire for the longitudinal velocity component; $S$, area of the geometrical contours; $t$, time.

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# DETERMINATION OF THE VELOCITY PROFILE OF A STREAM OF NON-NEWTONIAN FLUID


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We describe a method for determining the velocity profile from measured values of the flow rate in several pipes of various diameters.


The velocity distribution of a fluid over the cross section of a stream can be found rather simply for a known law of flow, i.e., the dependence of the flow velocity on the shear stress. Non-Newtonian fluids differ widely in their rheological behavior. The law of flow is an empirical relation which must be sought separately for each case.

Kutateladze et al. [1] succeeded in separating a rather large subclass of non-Newtonian fluids whose rheological behavior is fairly well described by the linear fluidity law

$$\varphi = \varphi_0 (1 + \vartheta\tau). \quad (1)$$

Smol'skii et al. [2] proposed to plot the velocity profile by graphical integration of $dw/dr = f(\tau_R)$, which in turn can be obtained from the Mooney formula by graphical differentiation of the experimental curve of the flow rate as a function of the wall friction stress. The velocity profile for practically any non-Newtonian fluid can be obtained by this method, but it is rather complicated and not very accurate.

The existing methods of measuring flow velocities (photokinetic, electrodiffusion, fluoroscopic, etc.) are complicated both with respect to procedure and with respect to the apparatus used.

We describe a simple method of plotting the velocity profile of the laminar flow of a fluid in a circular pipe. This method is based on the fact that for the same longitudinal pressure gradient the velocity curve in a channel of radius $R_1$ has the same shape as the portion of the curve for $r \leqslant R_1$ in a larger pipe of radius $R_2$ (Fig. 1). This conclusion follows from p. 67 of [2].

If the mean flow velocities $\overline{w}_i$ in N pipes of radii $r_i \leqslant R$ are measured for the same longitudinal pressure gradient, the piecewise linear approximation of the curve (Fig. 2) leads to the following relation for the velocity increment between the i-th and (i − 1)-th cross section of the profile:

$$\Delta w_i \approx 3\,\frac{\overline{w}_i r_i^2 - \overline{w}_{i-1} r_{i-1}^2}{r_i^2 + r_i r_{i-1} + r_{i-1}^2}$$

or

$$\Delta w_i \approx 3\,\frac{\overline{w}_i \beta_i^2 - \overline{w}_{i-1}}{\beta_i^2 + \beta_i + 1}, \quad (2)$$

where $\beta_i = r_i/r_{i-1}$. Then the true flow velocity $w_n$ at a distance $r_n$ from the pipe axis is given by the sum


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